UNIT 7 – RIGHT ANGLE TRIANGLES

Assignment	Title	Work to complete	Complete
	Vocabulary	Complete the vocabulary words on the attached handout with information from the booklet or text.	
1	***Triangles***	Labelling Triangles	
2	***Pythagorean Theorem***	Pythagorean Theorem	
3	***Trigonometry***	Trigonometry	
4	***The Sine Ratio***	The Sine Ratio	
	Angle of Elevation and Depression		
5	***Using Sine Ratio in Solving Right Triangles***	Using Sine Ratio in Solving Right Triangles	
6	***The Cosine Ratio***	The Cosine Ratio	
7	***Using Cosine Ratio in Solving Right Triangles***	Using Cosine Ratio in Solving Right Triangles	
8	***The Tangent Ratio***	The Tangent Ratio	
9	***Using Tangent Ratio in Solving Right Triangles***	Using Tangent Ratio in Solving Right Triangles	
10	***Finding Angles***	Finding Angles	
11	***Solving Sight Triangles***	Solving Sight Triangles	
Mental Math	**Mental Math** Non-calculator practice	Get this page from your teacher	
Practice Test	**Practice Test** How are you doing?	Get this page from your teacher	
Self-Assessment	**Self-Assessment** ***"Traffic Lights"***	On the next page, complete the self-assessment assignment.	
Chapter Test	**Chapter Test** Show me your stuff!		

Traffic Lights

In the following chart, decide how confident you feel about each statement by sticking a red, yellow, or green dot in the box. Then discuss this with your teacher ***BEFORE*** you write the test.

Statement	Dot
After completing this chapter;	
• I can use the Pythagorean theorem to calculate the missing side of a right triangle	
• I know when to choose sine (sin), cosine (cos) or tangent(tan) based on the information given	
• I can use the three basic trigonometric functions (sin, cos, tan) to find a missing side or angle of a right triangle	
• I can determine places in the workplace where I could us trigonometry	

Vocabulary: Unit 7

Right Angle Triangles

<table>
<tr>
<td>budget
*this term has been completed for you as an example</td>
<td>Definition

an estimate of the amount of money to be spent on a specific project or over a given time frame</td>
<td>Diagram: A sample of a personal monthly budget:
<table>
<tr><th>Net Pay</th><th></th><th></th><th>$2500</th></tr>
<tr><td>Rent</td><td>$600</td><td>Recreation</td><td>$100</td></tr>
<tr><td>Telephone</td><td>$75</td><td>Personal Care</td><td>$100</td></tr>
<tr><td>Utilities</td><td>$75</td><td>Savings</td><td>$150</td></tr>
<tr><td>Food</td><td>$500</td><td>Spending (CDs...)</td><td>$200</td></tr>
<tr><td>Transportation</td><td>$500</td><td>Other expenses</td><td>$100</td></tr>
<tr><td>Clothing</td><td>$100</td><td></td><td></td></tr>
<tr><td>Total</td><td></td><td></td><td>$2,500</td></tr>
</table>
</td>
</tr>
<tr><td>angle of depression</td><td>Definition</td><td>Diagram/Example</td></tr>
<tr><td>angle of elevation</td><td>Definition</td><td>Diagram/Example</td></tr>
<tr><td>cosine</td><td>Definition</td><td>Diagram/Example</td></tr>
<tr><td>hypotenuse</td><td>Definition</td><td>Diagram/Example</td></tr>
</table>

leg	Definition	Diagram/Example
Pythagorean theorem	Definition	Diagram/Example
right triangle	Definition	Diagram/Example
sine	Definition	Diagram/Example
tangent	Definition	Diagram/Example

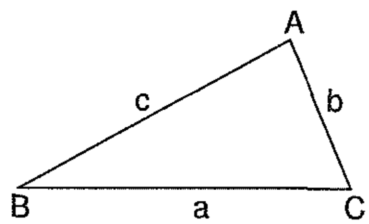
TRIANGLES

In this unit, you will be looking at triangles, specifically right angle triangles, also called right triangles. You will learn about Pythagorean Theorem and the basic trigonometric ratios. But first it is necessary to start with some facts about triangles.

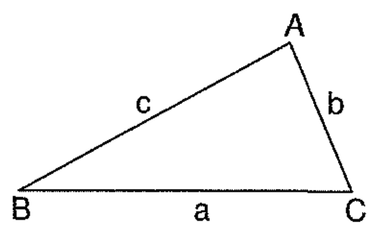
Fact 1: Every triangle contains 3 sides and 3 angles or vertices (plural of vertex).

Fact 2: The measurements of these angles always total 180^0. Remember this from the last unit??

Fact 3: To identify the side or vertex in a triangle, it is important to label the triangle following a standard routine. Each vertex of a triangle is labeled with a capital case letter – like "A" - and each side is labeled with the lower case letter that matches the opposite vertex. An example is below.



Another way to label the sides is with the capital letters of the two vertices the side connects. An example is below.

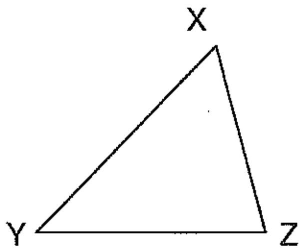


Side *a* can be called BC.
Side b can be called AC.
Side *c* can be called AB.

ASSIGNMENT 1 – LABELLING TRIANGLES

1) Label each side of the triangles below using a single lower case letter matching the opposite vertex.

a)



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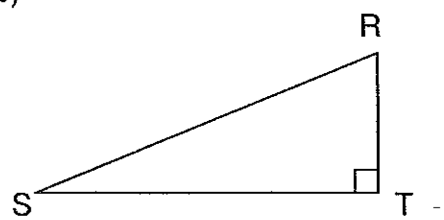

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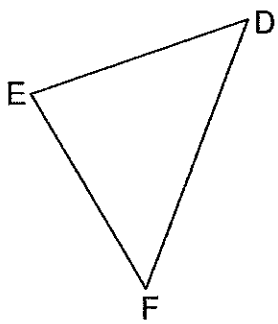

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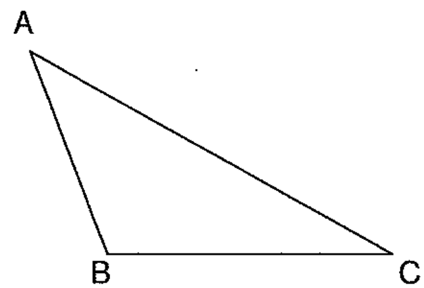
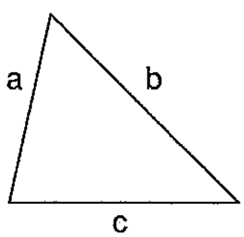


2) Label each vertex of the triangles below using a single capital letter matching the opposite side.

a)



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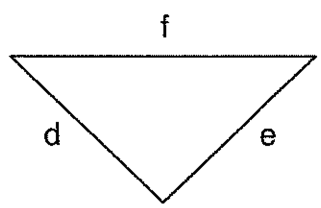
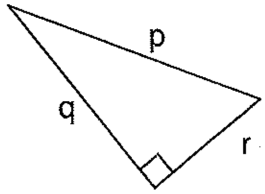
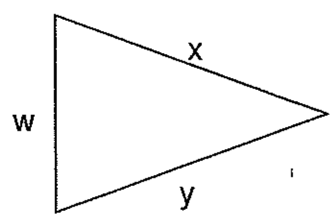


c)



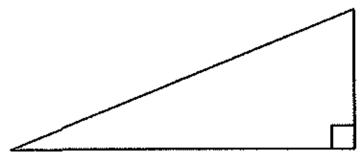
d)



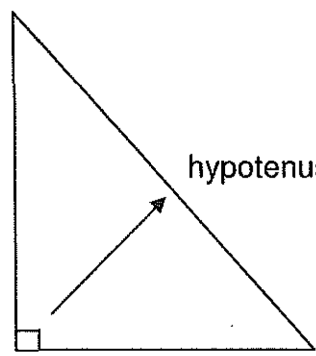
PYTHAGOREAN THEOREM

Pythagorean Theorem states the relationship between the sides of a right triangle. So, more facts about triangles are necessary.

Fact 4: A triangle that contains a 90^0 angle (a right angle) is called a right triangle (or right-angle triangle).



Fact 5: The side of the triangle that is opposite the 90^0 angle is always called the **hypotenuse**. It is labelled in the triangle below. The other two sides of the triangle are called legs.

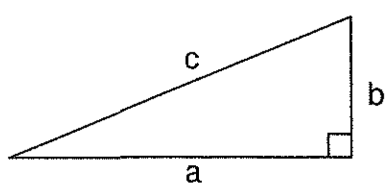


Fact 6: The hypotenuse is always the longest side in the triangle. It is always opposite the largest angle which is the 90^0 or right angle.

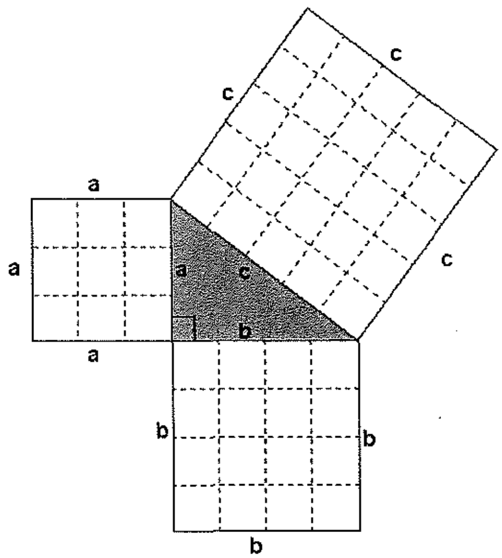
Fact 7: Pythagorean Theorem states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So in ΔABC with the right angle at C, the following relationship is true:

$$c^2 = a^2 + b^2$$

where a and b are the other 2 legs of the triangle.



Often Pythagorean Theorem is illustrated as the square of the sides as follows:



Notice that the length of side a is 3 boxes, side b is 4 boxes, and side c is 5 boxes. So if we calculate the area of each square, the following is true:

$c \times c = c^2 = 5 \times 5 = 25$
$b \times b = b^2 = 4 \times 4 = 16$
$a \times a = a^2 = 3 \times 3 = 9$

And we know that

$c^2 = a^2 + b^2$

So $25 = 9 + 16$ which is a true statement!

We can also rearrange the equation to find the length one of the legs;

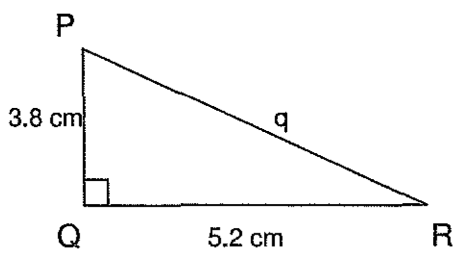
$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$b^2 = c^2 - a^2$

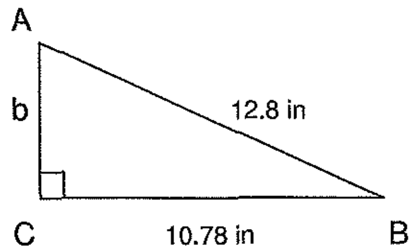
When we use Pythagorean Theorem to find a length of the hypotenuse or a leg, you need to have a calculator that has the square root function $\sqrt{\ }$ on it. The computer symbol looks like this: √ or $\sqrt{\ }$

<u>Example 1</u>: Use Pythagorean Theorem to find the length of the missing side to one decimal place.



Solution: $q^2 = p^2 + r^2$
$q^2 = 5.2^2 + 3.8^2$
$q^2 = 27.04 + 14.44$
$q^2 = 41.48$
$\sqrt{q^2} = \sqrt{41.48}$
$q \approx 6.44$ cm
Side q is approximately 6.4 cm

Example 2: Use Pythagorean Theorem to find the length of the missing side to one decimal place.



Solution: $c^2 = a^2 + b^2$

So, $b^2 = c^2 - a^2$

$b^2 = 12.8^2 - 10.78^2$

$b^2 = 163.84 - 116.21$

$b^2 = 47.66$

$\sqrt{b^2} = \sqrt{47.66}$

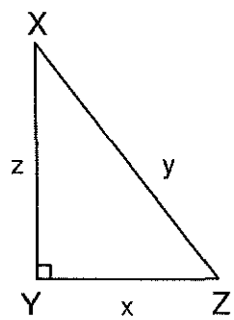
$b \approx 6.90$ cm

Side b is approximately 6.9 cm

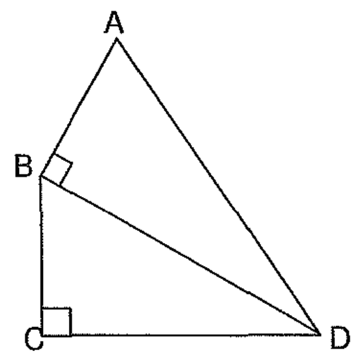
ASSIGNMENT 2 – PYTHAGOREAN THEOREM

1) Using the following triangles, use lettering provided to state the Pythagorean relations that apply.

a)



b)



2) Rearrange the following Pythagorean statement to solve for the other two legs.

$d^2 = e^2 + f^2$

3) A ladder is leaned against a house. The base of the ladder is *d* feet away from the house. Draw a diagram and then write the Pythagorean relationship that exists for these lengths. Use ℓ for the ladder, *h* for the house, and *d* for the distance the ladder is from the house. You are not required to solve this question.

4) A 40 foot ladder reaches 38 feet up the side of a house. How far from the side of the house is the base of the ladder? Draw a diagram and show your work.

5) A ramp into a house rises up 3.5 meters over a horizontal distance of 10.5 meters. How long is the ramp? Draw a diagram and show your work.

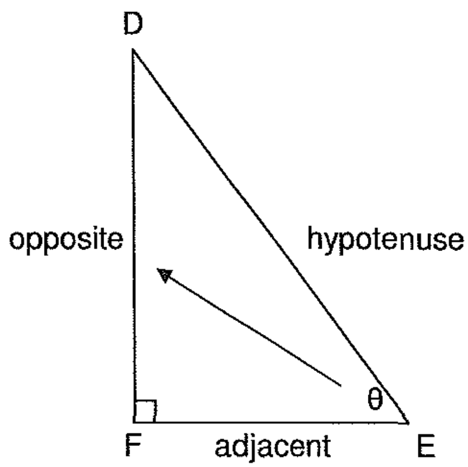
TRIGONOMETRY

Trigonometry is one of the most important topics in mathematics. Trigonometry is used in many fields including engineering, architecture, surveying, aviation, navigation, carpentry, forestry, and computer graphics. Also, until satellites, the most accurate maps were constructed using trigonometry.

The word ***trigonometry*** means triangle measurements. It is necessary to finish our triangle facts here.

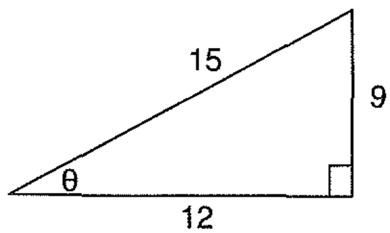
Fact 8: In trigonometry, the other two sides (or legs) of the triangle are referred to as the **opposite** and **adjacent** sides, depending on their relationship to the angle of interest in the triangle.

In this example, if we pick angle DEF – the angle labelled with the Greek letter θ – then we are able to distinguish the sides as illustrated in the diagram below.



The side that is opposite the angle of interest, in this case θ, is called the **opposite** side. The side that is nearest to angle θ and makes up part of the angle is called the **adjacent** side. To help you, remember that adjacent means beside. Although the hypotenuse occupies one of the two adjacent positions, it is **never** called the adjacent side. It simply remains the hypotenuse. This is why it is identified first. It is recommended to label the side in the order hypotenuse, opposite, and finally adjacent. You may use initials for these side, h, o, and a, but always use lower case letters to avoid mixing up the labelling with a vertex.

Example 1: Using the triangle below, answer the questions.



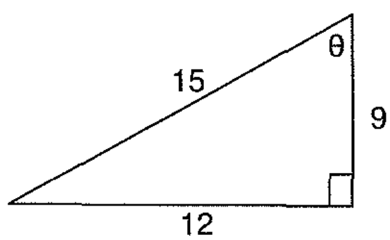
1) What is the hypotenuse? __________
2) What is the opposite side to θ? __________
3) What is the adjacent side to θ? __________

Solution:

1) What is the hypotenuse? 15
2) What is the opposite side to θ? 9
3) What is the adjacent side to θ? 12

This example uses the same triangle as in Example 1; however, this time, the *other* acute angle is labelled as θ. This is done to show that the opposite and adjacent sides switch when the other angle is the angle of interest. The hypotenuse **always** stays the same.

Example 2: Using the triangle below, answer the questions.



1) What is the hypotenuse? __________
2) What is the opposite side to θ? __________
3) What is the adjacent side to θ? __________

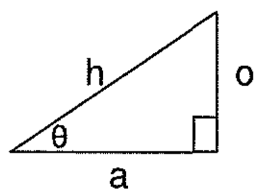
Solution:

1) What is the hypotenuse? 15
2) What is the opposite side to θ? 12
3) What is the adjacent side to θ? 9

ASSIGNMENT 3 – TRIGONOMETRY

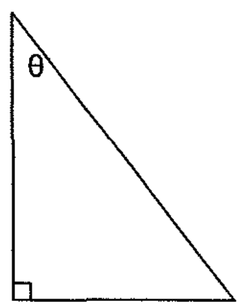
For each of the right triangles below, mark the hypotenuse, and the sides that are opposite and adjacent sides to θ as shown in the example.

Example:

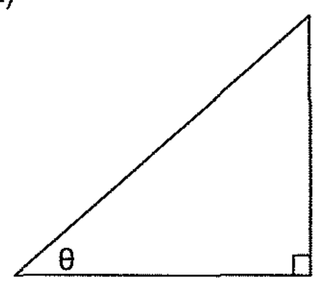


h = hypotenuse
o = opposite
a = adjacent

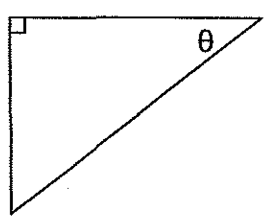
1)



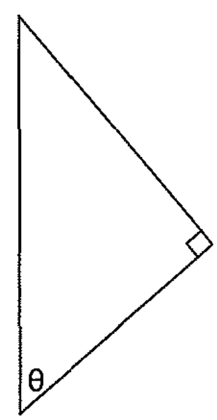
2)



3)



4)



TRIGONOMETRIC RATIOS

In the previous unit about similar figures, you learned that the ratios of corresponding sides of similar triangles are equal. When the angles of different triangles are the same, the ratio of the sides within the triangle will always be the same. They depend only on the measure of the angle of interest, not the size of the triangle. These ratios are the trigonometric ratios.

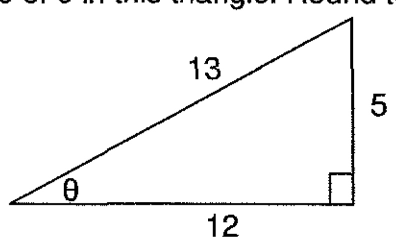
There are three trigonometric ratios we are concerned with: sine, cosine, and tangent.

THE SINE RATIO

The *sine of angle* θ means the ratio of the length of opposite side to the length of the hypotenuse. It is abbreviated as **sin θ** but read as sine θ. It is written like this:

$$\sin\theta = \frac{opposite}{hypotenuse} \quad \text{or} \quad \sin\theta = \frac{o}{h}$$

Example 1: Find the sine of θ in this triangle. Round to 4 decimal places.



Solution:

The opposite side is 5 and the hypotenuse is 13. So

$$\sin\theta = \frac{o}{h} = \frac{5}{13} = 0.3846 \qquad \text{So } \sin\theta = 0.3846$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following sine ratios. Round to 4 decimal places.

a) $\sin 15^0$ b) $\sin 67^0$ c) $\sin 42^0$

***** **REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG)** ****

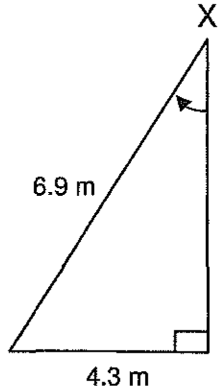
Solution: Type "sin" followed by the angle, and then "=" to solve

a) $\sin 15^0 = 0.2588$ b) $\sin 67^0 = 0.9205$ c) $\sin 42^0 = 0.6691$

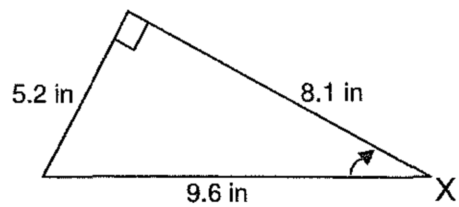
ASSIGNMENT 4 – THE SINE RATIO

1) Calculate the value of sin X to two decimal places.

a)



b)



2) Use your calculator to determine the value of each of the following sine ratios to four decimal places.

a) sin 10^0 = ______________

b) sin 48^0 = ______________

c) sin 77^0 = ______________

d) sin 85^0 = ______________

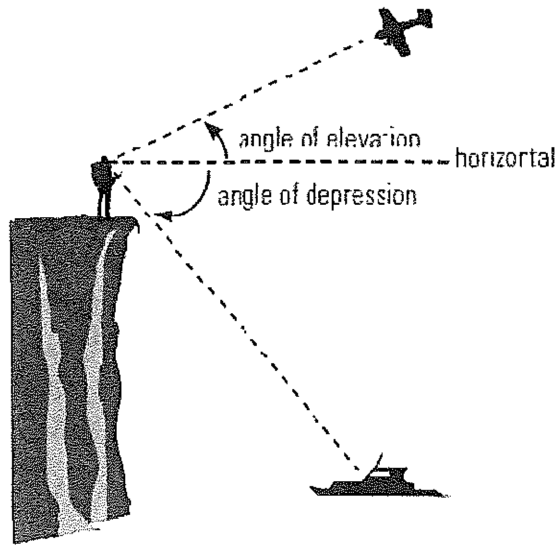
3) There are two special sine ratios. Calculate the following and suggest why the values are what the results give you.

a) sin 0^0 = ______________

b) sin 90^0 = ______________

ANGLE OF ELEVATION AND DEPRESSION

When you look up at an airplane flying overhead for example, the angle between the horizontal and your line of sight is called the **angle of elevation**.

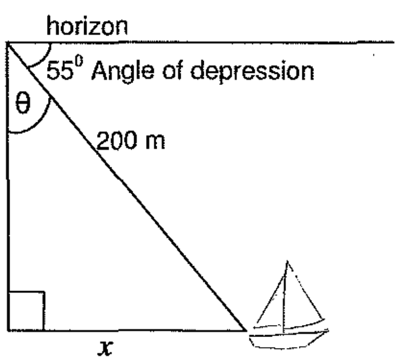


When you look down from a cliff to a boat passing by, the angle between the horizontal and your line of sight is called the **angle of depression**.

When you are given the angle of depression, it is important to carefully use this angle in your triangle.

Example 1: You are standing at the top of a cliff. You spot a boat 200 m away at an angle of depression of 55^0 to the horizon. How far is the boat from the coast? Draw a diagram to illustrate this situation.

Solution: Draw a diagram, label it with the information, and then solve the triangle.



The angle inside the triangle is the complement to the angle of depression. To find that angle, do the following:

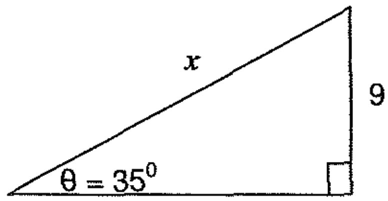
$\theta = 90^0 - 55^0$

$\theta = 35^0$

USING SINE RATIO IN SOLVING RIGHT TRIANGLES

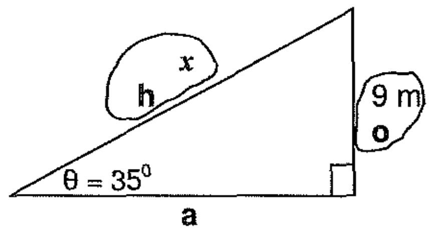
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The sine ratio can be used to find missing parts of a right triangle.

Example 1: Use the sine ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$$\sin \theta = \frac{o}{h}$$

$$\sin 35^0 = \frac{9}{x}$$

Step 4: Solve using the process Cross Multiply and Divide.

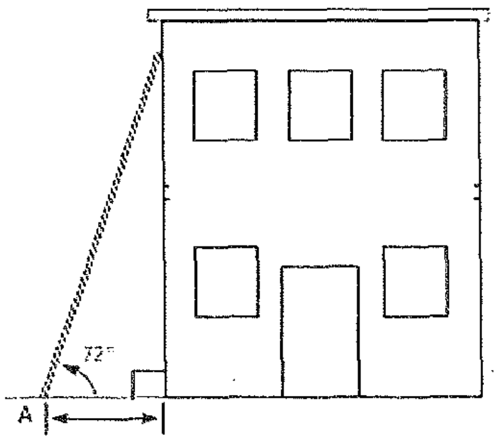
Since $\sin 35^0 = \frac{\sin 35}{1}$, then $\sin 35^0 = \frac{9}{x}$ becomes $\frac{\sin 35}{1} = \frac{9}{x}$

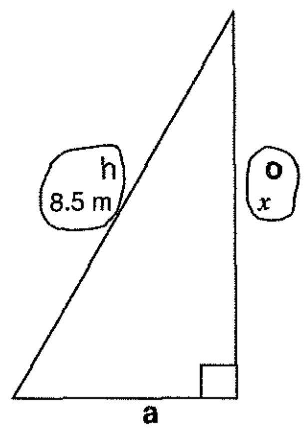
$x = 9 \times 1 \div \sin 35^0$
$= 9 \times 1 \div 0.5736$
$= 15.7$ m

Example 2: A ladder 8.5 m long makes an angle of 72^0 with the ground. How far up the side of a building will the ladder reach?

Solution:

Sketch a diagram and place the information from the question on this diagram. Remember that there will always be a right triangle in your diagram. It is often helpful to draw that triangle and copy the key information from the sketch.





Step 1: Label the sides of the triangle with **h**, **o** and **a**
See above right.

Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$$\sin\theta = \frac{o}{h}$$

$$\sin 72^0 = \frac{x}{8.5}$$

Step 4: Solve using the process Cross Multiply and Divide.

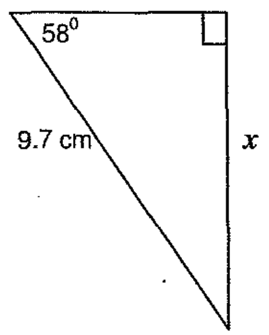
Since $\sin 72^0 = \frac{\sin 72}{1}$,then $\sin 72^0 = \frac{x}{8.5}$ becomes $\frac{\sin 72}{1} = \frac{x}{8.5}$

$x = \sin 72^0 \times 8.5 \div 1$
$= 0.9511 \times 8.5 \div 1$
$= 8.1$ m

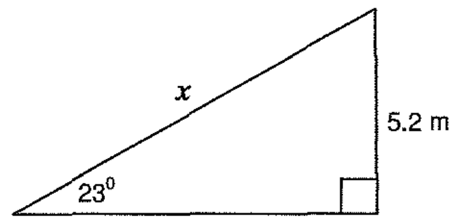
ASSIGNMENT 5 – USING SINE RATIO IN SOLVING RIGHT TRIANGLES

1) Calculate the length of the side indicated in the following diagrams.

a)



b)



2) A weather balloon with a 15 m string is tied to the ground. How high is the balloon if the angle between the string and the ground is 38^0?

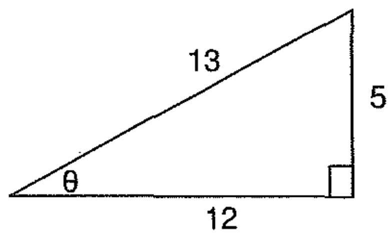
3) A ramp makes an angle of 22^0 with the ground. If the end of the ramp is 1.5 m above the ground, how long is the ramp?

THE COSINE RATIO

The co*sine of angle θ* means the ratio of the adjacent side to the hypotenuse. It is abbreviated as **cos θ** but read as cosine θ. It is written like this:

$$\cos\theta = \frac{adjacent}{hypotenuse} \quad \text{or} \quad \cos\theta = \frac{a}{h}$$

Example 1: Find the cosine of θ in this triangle.



Solution:

The adjacent side is 12 and the hypotenuse is 13. So

$$\cos\theta = \frac{a}{h} = \frac{12}{13} = 0.9231$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following cosine ratios. Round to 4 decimal places.

a) $\cos 15^0$ b) $\cos 67^0$ c) $\cos 42^0$

***** **REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG)** ****

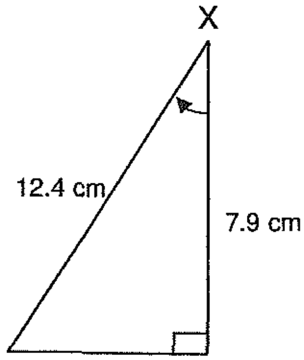
Solution: Type "cos" followed by the angle, and then "=" to solve

a) $\cos 15^0 = 0.9659$ b) $\cos 67^0 = 0.3907$ c) $\cos 42^0 = 0.7431$

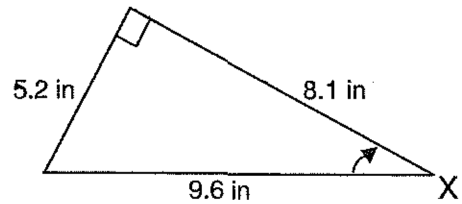
ASSIGNMENT 6 – THE COSINE RATIO

1) Calculate the value of cos X to two decimal places.

a)



b)



2) Use your calculator to determine the value of each of the following sine ratios to four decimal places.

a) cos 10^0 = ______________ b) cos 48^0 = ______________

c) cos 77^0 = ______________ d) cos 85^0 = ______________

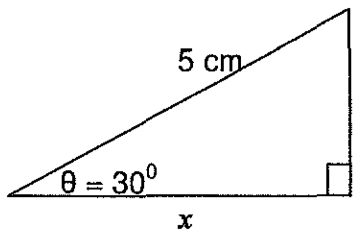
3) There are two special cosine ratios. Calculate the following and suggest why the values are what the results give you.

a) cos 0^0 = ______________ b) cos 90^0 = ______________

USING COSINE IN SOLVING RIGHT TRIANGLES

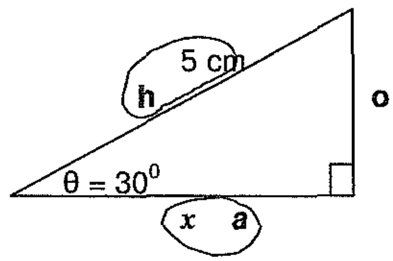
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The cosine ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h, o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **a** and **h** are being used, the correct ratio is **cos θ**

Step 4: Write down the chosen ratio and substitute the correct values into the correct ratio.

$$\cos\theta = \frac{a}{h}$$

$$\cos 30^0 = \frac{x}{5}$$

Step 5: Solve using the process Cross Multiply and Divide.

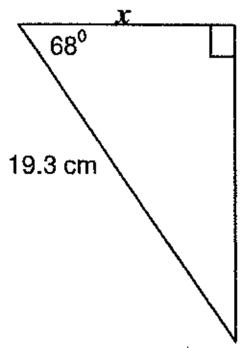
Since $\cos 30^0 = \dfrac{\cos 30}{1}$, then $\cos 30^0 = \dfrac{x}{5}$ becomes $\dfrac{\cos 30}{1} = \dfrac{x}{5}$

$x = \cos 30^0 \times 5 \div 1$
$= 0.8660 \times 5 \div 1$
$= 4.3$ cm

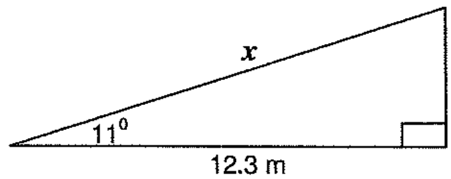
ASSIGNMENT 7 – USING COSINE RATIO IN SOLVING RIGHT TRIANGLES

1) Calculate the length of the side indicated in the following diagrams.

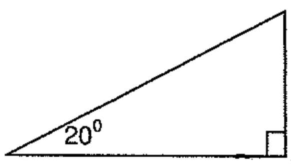
a)



b)



2) A child's slide rises to a platform at the top is 20^0. If the horizontal distance that the slide covers is 25 m long, how long is the slide?



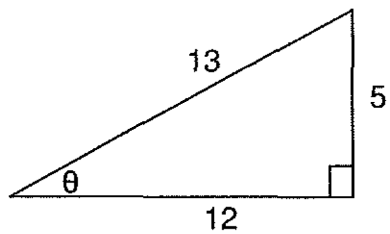
3) A flagpole is anchored to the ground by a guy wire that is 12 m long. The guy wire makes and angle of 63^0 with the ground. How far from the base of the flagpole must the guy wire be anchored into the ground?

THE TANGENT RATIO

The tangent *of angle θ* means the ratio of the opposite side to the adjacent side. It is abbreviated as **tan θ** but read as tangent θ. It is written like this:

$$\tan\theta = \frac{opposite}{adjacent} \quad \text{or} \quad \tan\theta = \frac{o}{a}$$

Example 1: Find the tangent of θ in this triangle.



Solution:

The opposite side is 5 and the adjacent side is 12. So

$$\tan\theta = \frac{o}{a} = \frac{5}{12} = 0.4167$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following tangent ratios. Round to 4 decimal places.

a) $\tan 15^0$ b) $\tan 67^0$ c) $\tan 42^0$

***** **REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG)** ****

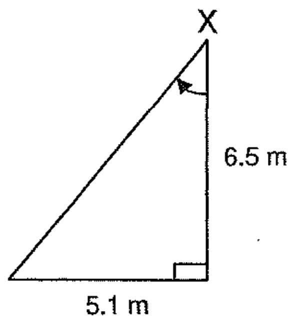
Solution: Type "tan" followed by the angle, and then "=" to solve

a) $\tan 15^0 = 0.2679$ b) $\tan 67^0 = 2.3559$ c) $\tan 42^0 = 0.9004$

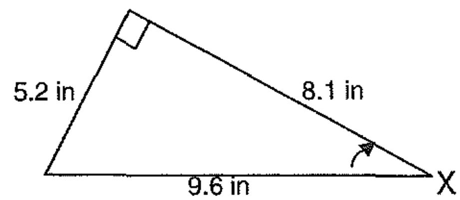
ASSIGNMENT 8 – THE TANGENT RATIO

1) Calculate the value of tan X to two decimal places.

a)



b)



2) Use your calculator to determine the value of each of the following sine ratios to <u>four</u> decimal places.

a) tan 10^0 = ______________ b) tan 48^0 = ______________

c) tan 77^0 = ______________ d) tan 85^0 = ______________

3) There are some special tangent ratios. Calculate the following and suggest why the values are what the results give you.

a) tan 0^0 = ______________

b) tan 45^0 = ______________

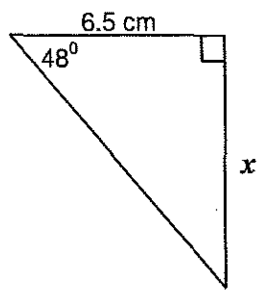
c) tan 89^0 = ______________

d) tan 90^0 = ______________

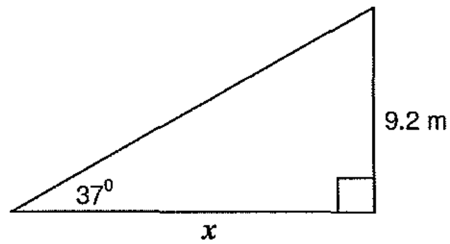
ASSIGNMENT 9 – USING TANGENT RATIO IN SOLVING RIGHT TRIANGLES

1) Calculate the length of the side indicated in the following diagrams.

a)



b)



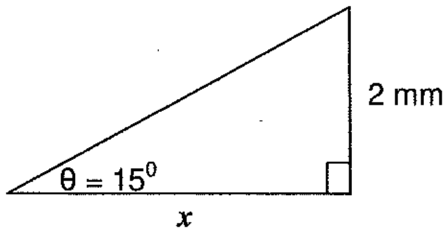
2) A man stands 12 m from the base of a tree. He views the top of the tree at an angle of elevation of 58^0. How tall is the tree?

3) How far from the side of a house is the base of a ladder if the angle of elevation is 70^0 and the ladder reaches 15 feet up the side of the house?

USING TANGENT IN SOLVING RIGHT TRIANGLES

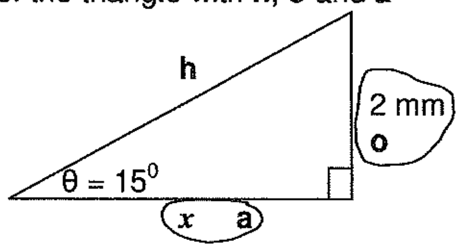
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The tangent ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h, o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **a** are being used, the correct ratio is **tan θ**

Step 3: Substitute the correct values into the correct ratio.

$\tan \theta = \frac{o}{a}$

$\tan 15 = \frac{2}{x}$

Step 4: Solve using the process Cross Multiply and Divide.

Since $\tan 15^0 = \frac{\tan 15}{1}$, then $\tan 15^0 = \frac{2}{x}$ becomes $\frac{\tan 15}{1} = \frac{2}{x}$

$x = 2 \times 1 \div \tan 15^0$
$= 2 \times 1 \div 0.2679$
$= 7.5$ mm

FINDING ANGLES

So far in this unit, you have used the trigonometric ratios to find the length of a side. But if you know the trigonometric ratio, you can calculate the size of the angle. This requires and "inverse" operation. You can use your calculator to find the opposite of the usual ratio provided you can calculate the ratio. To do this you need 2 sides in the triangle. You can think of the inverse in terms of something simpler: addition is the opposite or inverse of subtraction. In the same way, trig functions have an inverse.

To calculate the inverse, you usually use a 2nd function and the sin/cos/tan buttons on your calculator in sequence. If you look at your calculator just above the sin/cos.tan buttons, you should see the following: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$. These are the inverse functions. If you use these buttons, you will be able to turn a ratio into an angle.

Example 1: Calculate each angle to the nearest whole degree.

a) $\sin X = 0.2546$

b) $\cos Y = 0.1598$

c) $\tan Z = 3.2785$

Solution: Use the appropriate inverse function on your calculator.

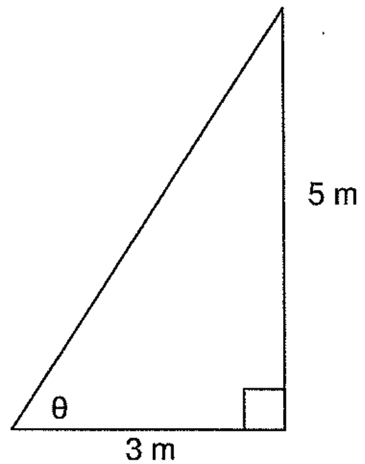
NOTE: Every calculator is different in how the buttons are keyed in order to achieve the desired outcome. Most calculators will need to key "2ndF sin" in order to get $\sin^{-1}$ displayed. Then key in the value with or without brackets as necessary.

a) $\sin X = 0.2546$
$X = \sin^{-1}(0.2546)$
$X = 14.74988^0$ Angle X is 15^0.

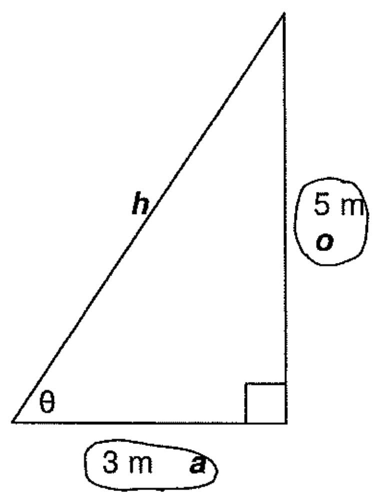
b) $\cos Y = 0.1598$
$Y = \cos^{-1}(0.1598)$
$Y = 80.8047^0$ Angle Y is 81^0.

c) $\tan Z = 3.2785$
$Z = \tan^{-1}(3.2785)$
$Z = 73.03737^0$ Angle Z is 73^0.

Example 2: Determine the angle θ in the following triangle.



Solution:



1) h, o, a the triangle

2) Circle the letters with their partner numbers

3) Choose the appropriate trig ratio. In this case, it is tangent.

4) Write down the ratio and fill it in.

$$\tan \theta = \frac{o}{a}$$

$$\tan \theta = \frac{5}{3}$$

5) Divide the numerator by the denominator in the fraction to get a decimal number.

$\tan \theta = 1.66666$

6) Use the inverse function to solve for θ.

$\theta = \tan^{-1}(1.66666)$
$\theta = 59.0352^0$ Angle θ is approximately 59^0.

ASSIGNMENT 10 – FINDING ANGLES

1) Calculate the following angles to the nearest whole degree.

a) sin D = 0.5491

b) cos F = 0.8964

c) tan G = 2.3548

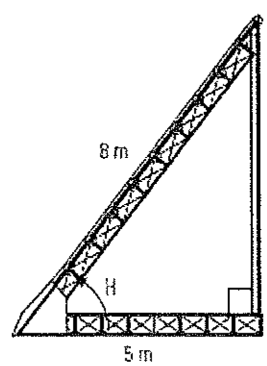
d) sin P = 0.9998

e) cos Q = 0.3907

f) tan R = 0.4663

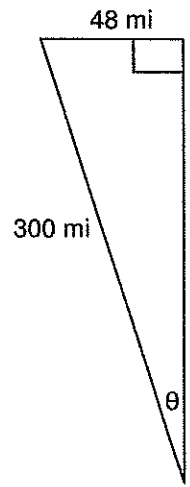
2) In a right triangle, ΔXYZ, the ratio of the opposite side to ∠X to the hypotenuse is 7:8 or $\frac{7}{8}$. What is the approximate size of ∠X?

3) At what angle to the ground is an 8 m long conveyor belt if it is fastened 5 m from the base of the loading ramp?

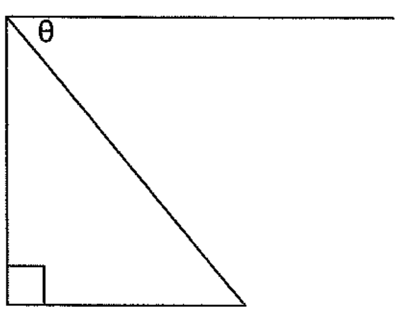


4) If a boat is 150 m from the base of a 90 m cliff, what is the angle of elevation from the boat to the top of the cliff?

5) After an hour of flying, a jet has travelled 300 miles, but gone off course 48 miles west of its planned flight path. What angle, θ, is the jet off course?



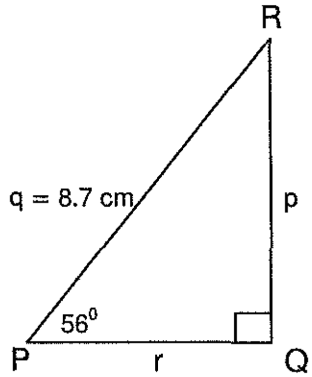
6) What is the angle of depression, θ, from the top of a 65 m cliff to an object 48 m from its base?



SOLVING RIGHT TRIANGLES

When asked to solve a right triangle, that means to find all the angle measures and the length of all the sides. Remembering that the angles in a triangle add up to 180^0, once two of the angles are known, the third can be calculated by subtraction. Also, once two of the sides are known, the third side can be found using Pythagorean Theorem – unless told not to use it! Then the third side should be found using trig ratios.

Example 1: Solve the right triangle below. Give lengths to the nearest tenth of a cm, and angles to the nearest whole degree.



Solution: Subtract to find the third angle, use trig to find side p, and use Pythagorean Theorem to find side r.

Part 1: $\angle R = 180 - 90^0 - 56^0$
$\angle R = 34^0$

Part 2: To solve for side p, use the sin ratio. Use $\angle$ P and the hypotenuse, 8.7 cm

$\sin P = \frac{o}{h}$

$\frac{\sin 56}{1} = \frac{p}{8.7}$

$p = \sin 56^0 \times 8.7 \div 1$

$p = 7.2$ cm

Part 3: Use Pythagorean Theorem to find side ***r***

$q^2 = p^2 + r^2$

$8.7^2 = 7.2^2 + r^2$

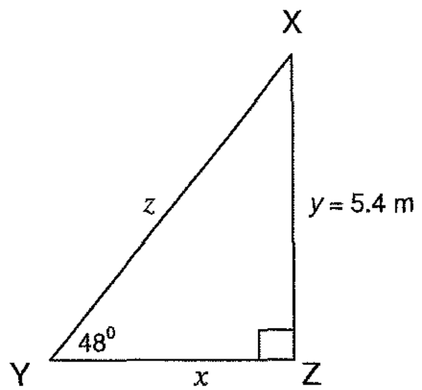
$r^2 = 8.7^2 - 7.2^2$

$r^2 = 23.85$

$\sqrt{r^2} = \sqrt{23.85}$

$r \approx 4.88$ cm $r \approx 4.9$ cm

Example 2: Solve the right triangle below without using Pythagorean Theorem. Give lengths to the nearest tenth of a cm, and angles to the nearest whole degree.



Solution: Subtract to find the third angle, use trig to find side x, and side z.

Part 1: $\angle X = 180 - 90^0 - 48^0$
$\angle R = 42^0$

Part 2: To solve for side x, use the tan ratio. Use $\angle$ Y and the side y, 5.4 m

$$\tan Y = \frac{o}{a}$$

$$\frac{\tan 48}{1} = \frac{5.4}{x}$$

$x = 1 \times 5.4 \div \tan 48^0$
$x = 4.9$ m

Part 3: To solve for side z, use the sin ratio. Use $\angle$ Y and the side y, 5.4 m

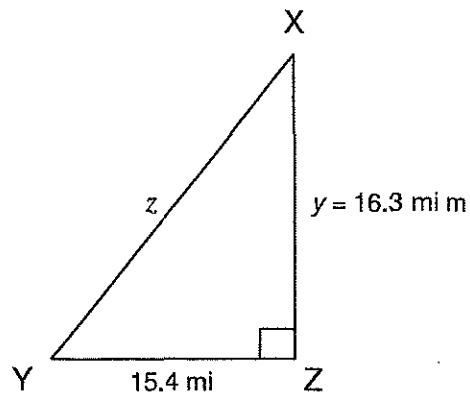
$$\sin Y = \frac{o}{h}$$

$$\frac{\sin 48}{1} = \frac{5.4}{z}$$

$z = 1 \times 5.4 \div \sin 48^0$
$z = 7.3$ m

Note: any trig ratio involving side z would work. These numbers were chosen because they are exact from the given information, and thus more accurate.

Example 3: Solve the right triangle below without using Pythagorean Theorem. Give lengths to the nearest tenth of a cm, and angles to the nearest whole degree.



Solution: Use trig to find ∠ Y, subtract to find the third angle, and use trig to find side z.

Part 1: To find ∠ Y, use the tan ratio. Use side x, 15.4 mi and the side y, 16.3 mi

$\tan Y = \frac{o}{a}$

$\tan Y = \frac{16.3}{15.4}$

$\tan Y = 1.0584$

$\theta = \tan^{-1}(1.0584)$
$\theta = 46.6263^0$ Angle θ is approximately 47^0.

Part 2: : $\angle X = 180 - 90^0 - 46^0$
$\angle R = 43^0$

Part 3: To solve for side z, use the sin ratio. Use ∠ Y and the side y, 5.4 m

$\sin Y = \frac{o}{h}$

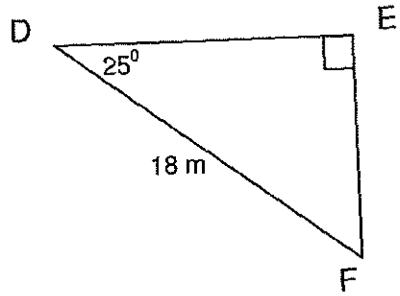
$\frac{\sin 43}{1} = \frac{16.3}{z}$

$z = 1 \times 16.3 \div \sin 43^0$
$z = 23.9$ mi

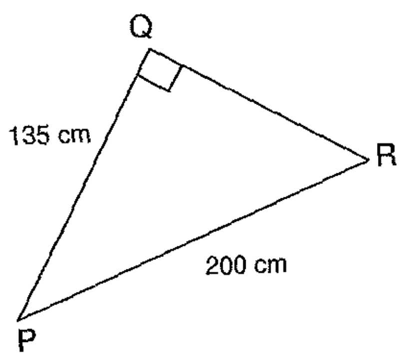
ASSIGNMENT 11 – SOLVING RIGHT TRIANGLES

1) Solve the given triangle.

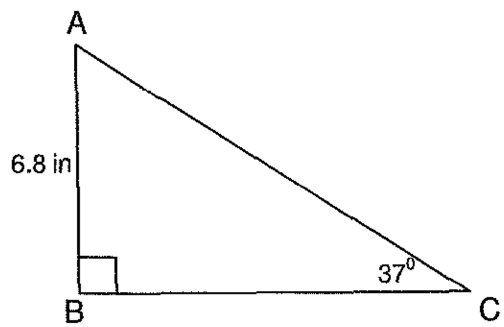
a)



b)



2) Solve the triangle below without using Pythagorean Theorem.



UNIT REVIEW

The techniques used to solve problems involving right angle triangles have widespread applications in many real-life areas. Often, word problems are presented where Pythagorean Theorem or trigonometry ratios are used to answer a question. The following section of questions requires that you solve these problems in that way. Remember these points to help you solve trig problems:

1. After reading the given information carefully, draw and label a diagram of a triangle if one is not provided.
2. When a side is the unknown and no angle is given, use the Pythagorean theorem to find the unknown side.
3. When a side is the unknown and an angle is given, use the appropriate trig ratio and only the sin, cos, or tan key on the calculator.
4. When an angle is the unknown, use the 2ndF key in addition to the sin, cos or tan key to get the inverse functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.
5. Label the triangle with o, a, and h in order to choose the correct trig ratio.
6. Solve for the unknown.
7. Check to see if your answer is reasonable.

Some students find it helpful to remember the trigonometric relationships of sine, cosine, and tangent with the phrase **SOH CAH TOA** (pronounced sow-a caw toe-a).This comes from the initials from the trig ratios and their sides:

$\sin = \frac{o}{h}$ $\cos = \frac{a}{h}$ $\tan = \frac{o}{a}$

You will be given the trigonometric ratios and Pythagorean Theorem in the Data Pages for both your tests and the Provincial exam. You are responsible for knowing how to use and apply these formulas.