

Integers

Introduction:

What is an integer?

What are some examples of integers in real life?

How could you draw a number line of integers?

How do we add and subtract integers? Try the following addition and/or subtraction questions? What rules can you discover? How could you do these questions on a number line?

1. $5 + 3 =$

5. $5 - 3 =$

2. $5 + (-3) =$

6. $5 - (-3) =$

3. $-5 + (-3) =$

7. $-5 - (-3) =$

4. $-5 + 3 =$

8. $-5 - 3 =$

Rules for adding and subtracting integers:

Try these ones using your rules.

1. $11 + 14 =$

5. $11 - 14 =$

2. $11 + (-14) =$

6. $11 - (-14) =$

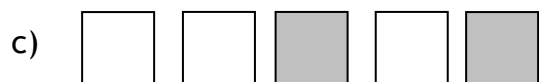
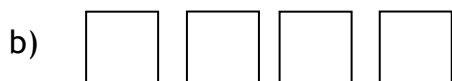
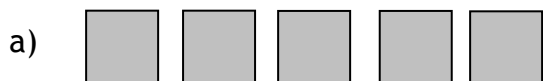
3. $-11 + (-14) =$

7. $-11 - (-14) =$

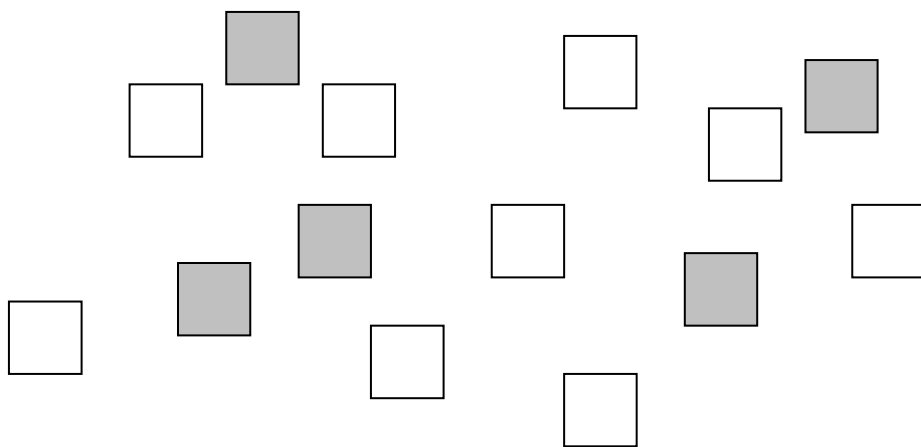
4. $-11 + 14 =$

8. $-11 - 14 =$

You may have used coloured tiles to count or group numbers. If the shaded tile represents -1 and the unshaded tile represents $+1$, what number does the following picture represent?



What number is this?

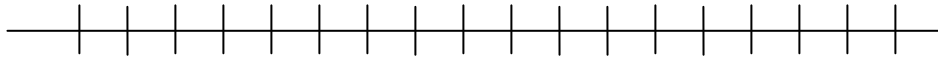


Note that a shaded tile and an unshaded tile cancel each other out or add up to zero. We call this a “zero pair.”



2.1 Using Models to Multiply Integers

We can think of multiplication as a shortcut for repeated addition. For example, it's way easier to do $4 \times 3 = 12$ than $3 + 3 + 3 + 3 = 12$. You can also think of it as jumping along a number line with a jump size of 3 each time.



2.2 Developing Rules to Multiply Integers

In the following table, multiply the number on the top of the column by the number in the start of the row. Work with the people in your group to ensure that you have the same numbers in each space. What patterns do you notice?

x	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
-5											
-4											
-3											
-2											
-1											
0											
+1											
+2											
+3											
+4											
+5											

Patterns for Multiplying Integers

1. Multiplying by zero
2. Multiplying by 1
3. Two positive integers multiplied
4. Two negative integers multiplied
5. One positive and one negative integer multiplied
6. Commutative Property
How does 3×4 compare to 4×3 ?
7. Distributive Property
Is $3 \times (2 + 4)$ the same as $3 \times 2 + 3 \times 4$?

Here's one way to remember patterns 5 to 8 (This seems to make more sense to girls.)

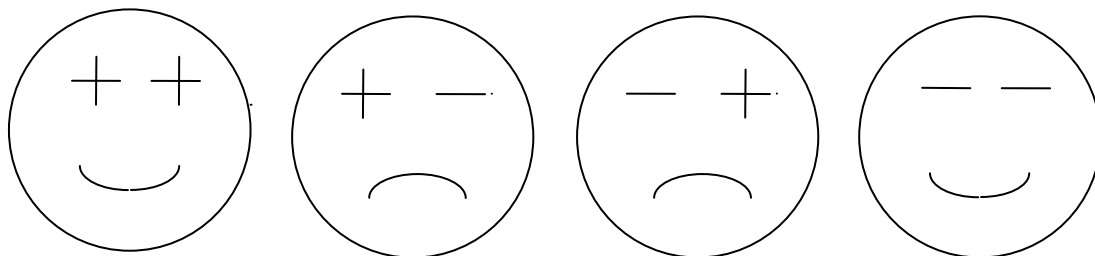
When a good thing happens to a good person, that's good.

When a bad thing happens to a bad person, that's good.

When a bad thing happens to a good person, that's bad.

When a good thing happens to a bad person, that's bad.

What do you think the following faces represent?



Word Problems:

1. Brian's mum was draining their swimming pool. The water level dropped 4 cm per hour for 11 hours. What was the total decrease in water level?
2. Lana was playing a board game with her friends. She had to move her playing piece back 3 spaces 4 times. How many spaces in total did she move?
3. Luca went up Grouse Mountain. The temperature at the base of the mountain was 14°C . As she went up the mountain, the temperature declined by 2° for every 1000 m. If he went up 4000 m, what was the temperature at the top?
4. The North Shore Rescue Team helped a man who had fallen into a ravine. If they needed 6 ropes of 20 m length per rope, how far did the rope go down into the ravine?

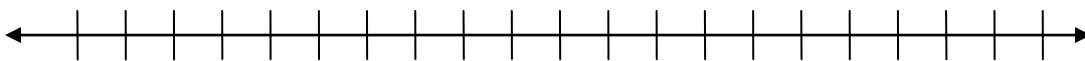
2.3 Using Models to Divide Integers

We can think of division as the opposite of multiplication. We can ask ourselves, "how many of the second number go into the first number?"

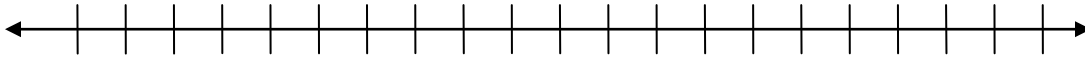
For example: $6 \div 2 = ?$

One can also picture integer division by walking along the number line. The first number is where you start and the second number is step size. Try the following examples.

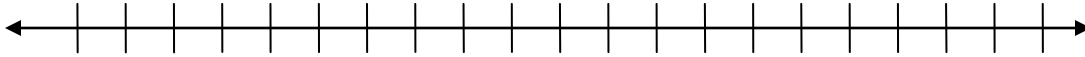
$$12 \div 2$$



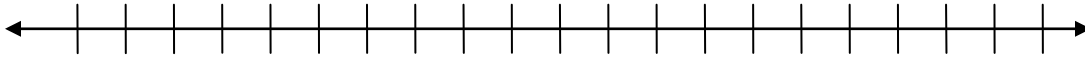
$$(-16) \div (-4)$$



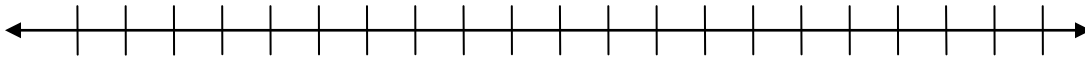
$$(-9) \div 3$$



$$10 \div (-5)$$



$$(-12) \div (-3)$$



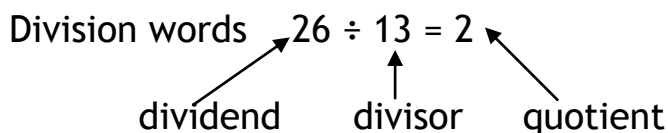
You don't have to use tiles or number lines to do integer division. Sometimes it's quickest just to remember the rules and apply them.

2.4 Developing Rules to Divide Integers

What are some rules that help when dividing integers?

1. A positive integer divided by a positive integer is
2. A positive integer divided by a negative integer is
3. A negative integer divided by a positive integer is
4. A negative integer divided by a negative integer is

Have you seen these patterns before?



In more “mathy” words, we could say that when the dividend and the divisor are both the same type of integer (either positive or negative) the quotient is positive. When we have one of each type of integer, the quotient is negative.

Try these.

1. $64 \div 8 =$
2. $27 \div (-9) =$
3. $(-16) \div 2 =$
4. $(-54) \div (-9) =$

Note that divisions can also be written as a fraction. Solve the following.

$$\frac{121}{-11} =$$

$$\frac{-72}{-9} =$$

$$\frac{-35}{5} =$$

2.5 Order of Operations with Integers

It’s really important that we do operations (things like adding, multiplying or using exponents) in the correct order. We often use the acronym BEDMAS to help us.

B

E

DM

AS

We write division/multiplication together and addition/subtraction together because we do these operations at the same time. Note that brackets can include things under a square root sign.

Here are some examples to try.

1. $10 + 12 \div 2^2 - 6 =$

2. $6^2 + (5 - 7) \times 3 =$

3. $17 - (-2) \times (-4) + (-5) =$

4. $\sqrt{25} + (-12) \div 6 + 11 \times 2 - (-3) =$

5. $\frac{4 + 2 \times 5}{12 - 5} =$