KEY

Mathematics 10
Chapter 7 Linear Equations and Graphs

Source: Mathematics 10
Class Handouts

Assignment/Homework and Quiz/Test Outline
(May be ***altered*** as the class progresses: approx.12 classes)

Section	Topic	Assignment / Homework	Date Assigned/Due
7.0 Activating Prior Knowledge	Review Linear Relations and Slope	APK-handout (in-class)	
7.1	Slope-Intercept Form	p349-355(1, 3ace, 4, 5, 6acef, 7, 8, 9, 10, 11, 12, 13, 15)	
7.2	General Form	p365-368 [1, 2, 3(omit graphs), 4, 5, 6, 7, 10, 11, 13ac, 14]	
Quiz 1	**Section 7.1-7.2**		
7.3	Slope-Point Form	p377-381 (1, 2, 3, 4a, 5, 6ace, 7, 8, 11, 12, 14, 17)	
7.4	Parallel and Perpendicular Lines	p390-394 (1 to 7, 9, 10, 11, 13, 14, 15, 16) In-class: "Mini" IB project (group work)	
Quiz 2	**Section 7.3-7.4**		
Quick Review		Review 7.1-7.4 Handout	
Chapter Review		p396-398 (all questions) (Optional)	
Practice Test		p309-401 (Optional)	
Chapter 7 Test		HW, Quick Review and Provincial Exam Question WS will be checked on the unit test day.	
IB Project	**What is a line?**	How can straight lines be used to create a design or composition? How are different equations involving slope related or connected to one another?	Due: TBA

Slope	Slope formula	Slope-Intercept	General Form	Slope-Point Form	Standard Form
$m = \frac{\text{rise}}{\text{run}}$	$m = \frac{y_2 - y_1}{x_2 - x_1}$	$y = mx + b$	$Ax + By + C = 0$	$y - y_1 = m(x - x_1)$	$Ax+By=D$

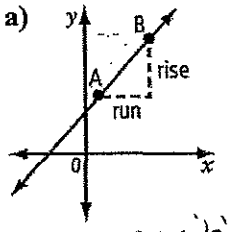
Math 10
Linear Equations and Graphs
Activating Prior Knowledge (APK)

Name_____
Date_____ Blk___

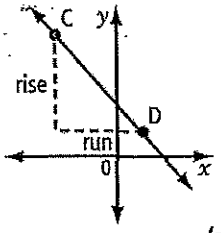
Review: Linear Relation and Slope

Slope = $\frac{\text{vertical change}}{\text{horizontal change}}$	OR	$m = \frac{\text{rise}}{\text{run}}$	OR	*Slope formula*	$m = \frac{y_2 - y_1}{x_2 - x_1}$

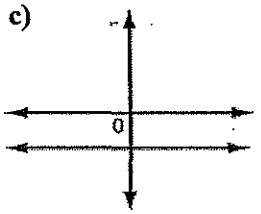
1. Slope of a line. Describe the slope, m, of each line

a) 

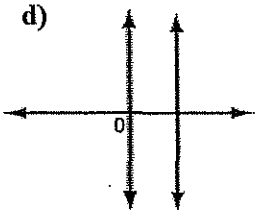
slope is positive

b) 

slope is negative

c) 

slope is zero

d) 

slope is undefined

2. For each linear relation, determine the pattern for the x and y variable and slope. Complete each table.

a)

Time(s)	Distance(m)	
1	4	
3	10	+6
5	16	+6
7	22	+6
		+2 +2 +2

Pattern increase 3m/s
Slope 6m/2s = 3m/s

b)

Time(s)	Distance(m)
0	-6
2	-11
4	-16
6	-22 -21

Pattern Decrease -2.5m/s
Slope -5m/2s = -2.5m/s

c)

Time(s)	Distance(m)
1	5
3	6
5	7
7	8

Pattern increase 0.5m/s
Slope 1m/2s = 0.5m/s

3. Identify the independent and dependent variable for each relation. Write an appropriate equation.

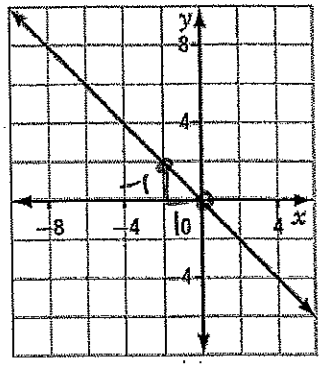
a) The student council rents a portable dunk tank for $250 for a school fund raising activity. Students pay $2 per ball, b, for the chance to hit a target and dunk a teacher into the tank. The total funds raised, f, will be used to support school activities. The independent variable is b and the dependent variable is f. An equation for this relation is: f = 2b − 250.

b) The cost, C, to take students to Science World is $80 to pre-book the field trip, and $11.25 per student, n. The independent variable is n and the dependent variable is C.
An equation for this relation is: C = 11.25n + 80.

c) The taxi fee, T, is $3.50 to start plus $1.75 for each kilometre, k, travelled.
The independent variable is k and the dependent variable is T.
An equation for this relation is: T = 1.75k + 3.50.

d) An oil delivery truck is filling the oil tank at John's house. The truck arrived with 3000 litres of oil. The number of litres of oil, l, remaining in the truck at t minutes decrease as at a rate of 80 litres per minute.
The independent variable is t and the dependent variable is l.
An equation for this relation is: l = 3000 − 80t.

4. Determine the slope of the line on the graph using both methods.



a) *slope* = $\frac{\text{rise}}{\text{run}}$

y-int: (0,0)

slope: $\frac{-1}{1}$ so -1

$y = -x$

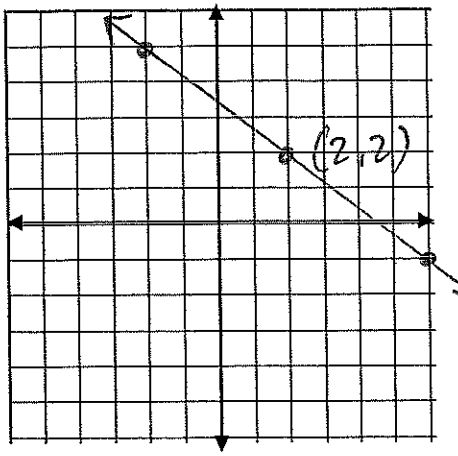
b) slope formula = $\frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{-1-0} = \frac{1}{-1} = -1$

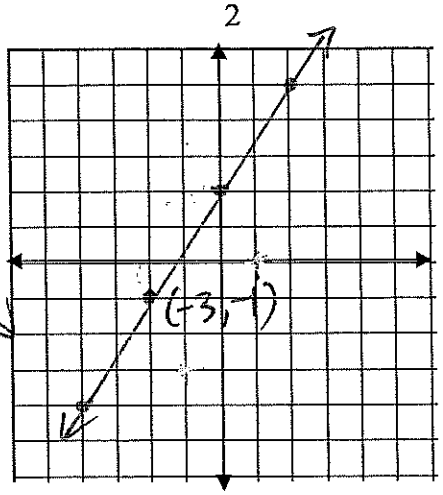
(0,0) and (-1,1)

5. Graph the line that passes through the given point and slope, *m*.

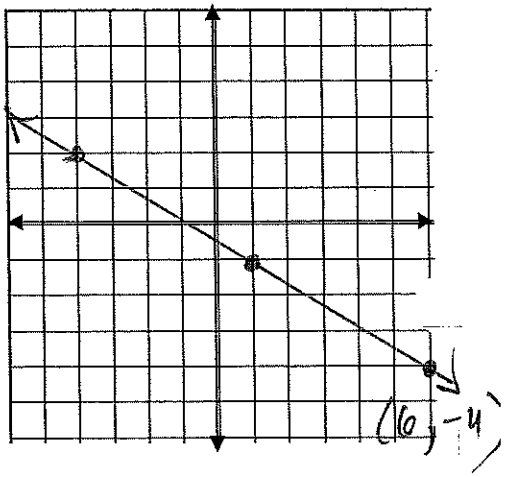
a) (2, 2); $m = \frac{-3}{4}$



b) (-3, -1); $m = \frac{3}{2}$



c) (6, -4); $m = \frac{-3}{5}$



6. Solve for ***k***, given the following information.

a) A line contains points A(-2, 4) and B(3, ***k***)
The slope of the line is -3/5.

$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{k-4}{3-(-2)} = \frac{-3}{5}$

$-3 = k - 4$

$\boxed{1 = k}$

b) A line contains points A(1,-11) and B(***k***, -3)
The slope of the line is $\frac{7}{4}$

$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{-3-(-11)}{k-1} = \frac{7}{4}$

$k - 1 = 4$

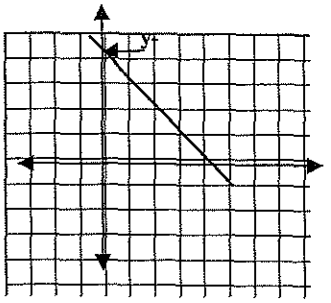
$\boxed{k = 5}$

Math 10
Linear Equations and Graphs
7.1 Slope-Intercept Form

Name_____
Date_____ Blk___

***y*-intercept**: the *y* coordinate of the point where a line or curve crosses the y-axis.
intercep(0,y)

To determine the y-intercept: If $x=0$, $y=?$ or the point $(0, y)$



Slope-intercept form: the equation of a line in the form $y = mx + b$.

Parameter: a variable that has a constant value in a particular equation.

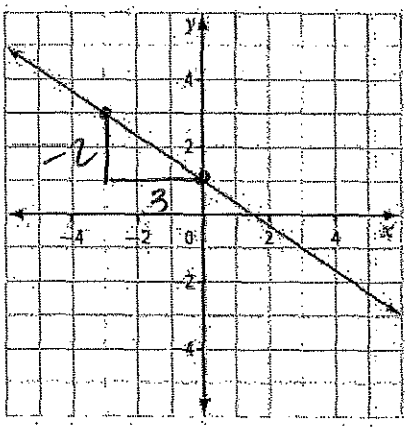
Slope-intercept form

$y = mx + b$ where m is the slope of the line and b is the y-intercept

slope y-intercept

Ex1. Using the graph, write the **equation** of a line in slope-intercept form, $y = mx + b$.

a)



Step 1. Find the slope of the line.

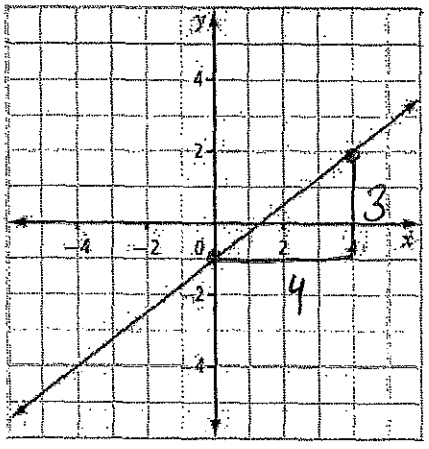
$m = \frac{rise}{run} = \frac{-2}{3}$

Step 2. Find the y-intercept. (0, 1)

Step 3. Write the equation of the line.

$y = \frac{-2}{3}x + 1$

b) Using the graph, write the equation in the slope-intercept form, $y = mx + b$.



The slope : $m = \frac{rise}{run} = \frac{3}{4}$

The y-int : (0, -1)

So $y = \frac{3}{4}x - 1$

Ex2. Express each equation in slope-intercept form, $y = mx + b$.
Determine the slope and y-intercept of each line. (Like solving an equation for y)

a) $4x + 2y = 12$

$2y = -4x + 12$

$y = \frac{-4}{2}x + 6$

$y = -2x + 6$

b) $12x - y = 8$

$12x - 8 = y$

c) $3x - 2y - 600 = 0$

$-2y = -3x + 600$

$y = \frac{3}{2}x - 300$

Ex3. Given slope, m, and the y-intercept, write the equation in the form $y = mx + b$.

a) $m = -6$; y-intercept = 4

$y = -6x + 4$

b) m = 0; y-intercept = 4/7

$y = \frac{4}{7}$

c) m = 2; y-intercept = –5

$y = 2x - 5$

Ex4. Given the equation $y = 4x + b$, and a point on the graph of a line, find b.

a) (2,-4) Sub this point into

$y = 4x + b$

$-4 = 4(2) + b$

$-4 = 8 + b \quad b = -12$

b) (-8, -5)

$y = 4x + b$

$-5 = 4(-8) + b$

$-5 = -32 + b$

$27 = b$

Ex5. Given the equation $y = mx - 5$, and a point on the graph of a line, find m.

a) (3,-4) Sub point into

$y = mx - 5$

$-4 = m(3) - 5$

$1 = 3m$

$\frac{1}{3} = m$

b) (-3, -5)

$y = mx - 5$

$-5 = -3m - 5$

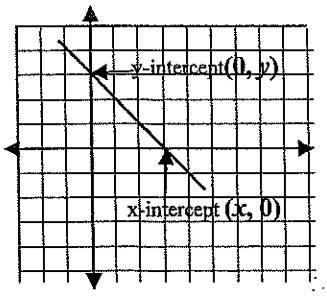
$0 = -3m$

$0 = m$ ← this means the line is horizontal

$y = -5$

Math 10
Linear Equations and Graphs
7.2 **General Form**

Name_____
Date_____ Blk___



x-intercept: the x-coordinate of the point where a line or curve crosses the x-axis
To determine the x-intercept: If $y=0$, $x=?$ or the "x" value in the point $(x, 0)$

General Form: the equation of a line in the form $Ax + By + C = 0$

- A, B, and C are real numbers
- A and B are not both zero
- By convention, A is a **whole number** and A is **positive**

Ex1. Convert slope-intercept form to the General Form.

a) $y = \frac{-2}{3}x + 6$

$\frac{2}{3}x + y - 6 = 0$

b) $y = \frac{3}{4}x - 2$

$0 = \frac{3}{4}x - y - 2$

c) $y = \frac{1}{4}x + \frac{2}{3}$

$0 = \frac{1}{4}x - y + \frac{2}{3}$

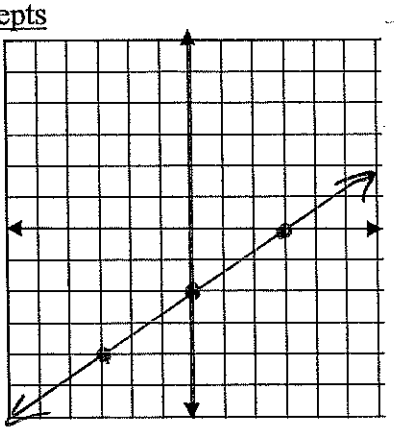
Ex2. Given the linear equation $2x - 3y - 6 = 0$, sketch the graph using intercepts

a) Find the **x-intercept** $(x, 0)$ by substituting $y = 0$ in the equation.

$2x - 3(0) - 6 = 0$
$2x - 6 = 0$
$2x = 6$
$\boxed{x = 3}$

b) Find the **y-intercept** $(0, y)$ by substituting $x = 0$ in the equation.

$2(0) - 3y - 6 = 0$
$-3y - 6 = 0$
$-3y = 6$
$\boxed{y = -2}$



Ex3. Given the following equation, find the x-intercept and y-intercept.

a) $y = 7x + 9$

xint:
$0 = 7x + 9$
$-9 = 7x$
$\frac{-9}{7} = x$

yint:
$y = 7(0) + 9$
$y = 9$

or just read from $y = 7x + 9$

b) $4x - 6y - 12 = 0$

xint:
$4x - 6(0) - 12 = 0$
$4x - 12 = 0$
$4x = 12$
$\boxed{x = 3}$

yint:
$4(0) - 6y - 12 = 0$
$-6y - 12 = 0$
$-6y = 12$
$\boxed{y = -2}$

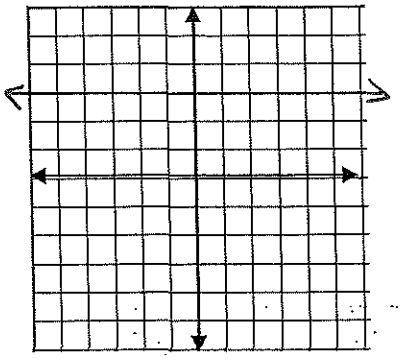
Ex4. Identify Intercepts of Horizontal or Vertical lines. Then sketch each linear relation.
State the domain and range of each line graph.

a) $y - 3 = 0$

$y = 3$

$\{x \mid x \in R\}$

$\{y \mid y = 3\}$

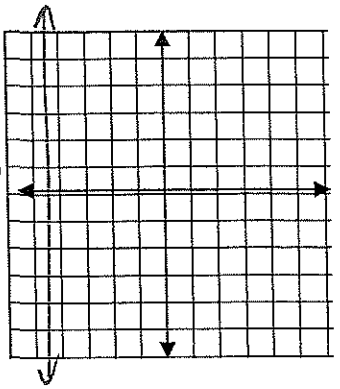


b) $x + 4.5 = 0$

$x = -4.5$

$\{x \mid x = -4.5\}$

$\{y \mid y \in R\}$



Ex5. Identify the value of the unknown parameter (variable) in the equation.
Substitute the values of x and y from the given point. Then solve for the unknown variable.

a) $Ax + 10y - 12 = 0$, passing through (-6, 4)

$A(-6) + 10(4) - 12 = 0$

$-6A + 40 - 12 = 0$

$-6A = -28$

$A = \frac{14}{3}$

b) $8x - 6y + C = 0$, passing through (-4, -6)

$8(-4) - 6(-6) + C = 0$

$-32 + 36 + C = 0$

$4 + C = 0$

$C = -4$

Problem Solving. Interpreting Intercepts

Ex5. Brooke wants to save $336. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns $12 per hour. On weeknights, she earns $16 per hour as a tutor.

a) Write an equation to represent the number of hours worked as a snowboard instructor, S, and as a tutor, T.

$336 = 12S + 16T \rightarrow 0 = 12S + 16T - 336$

b) Find the S-intercept. What does the S-intercept represent?

$336 = 12S + 16(0)$

$336 = 12S$ The time it takes to make $336 just as a snowboard instructor ✓

c) Find the T-intercept. What does the T-intercept represent?

$336 = 12(0) + 16T$

$336 = 16T$ The time it takes to make $336 just as a tutor

d) Suppose Brooke works 8 hours as a snowboard instructor. How many hours will she need to work as a tutor?

$336 = 12(8) + 16T$

$240 = 16T \rightarrow T = 15$ hours

Mathematics 10, p365-368 [1, 2, 3(omit graphs), 4, 5, 6, 7, 10, 11, 13ac, 14]

Math 10
Linear Equations and Graphs
7.3 **Slope-Point Form**

Name_____
Date_____ Blk___

Slope-Point form: the equation of a non-vertical line in the form of $y - y_1 = m(x - x_1)$
where m is the slope and (x_1, y_1) are the coordinates of a point on a line.

The slope-point form is developed from slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Ex1. **Use the Slope-Point form** to write the equation of a line, given a point on the line and the slope, m.

a) Given (–2, 5) and slope = –3

$y - y_1 = m(x - x_1)$

$y - 5 = m(x + 2)$

$y - 5 = -3(x + 2)$

b) Given (3,–4) and slope = 2

$y - y_1 = m(x - x_2)$

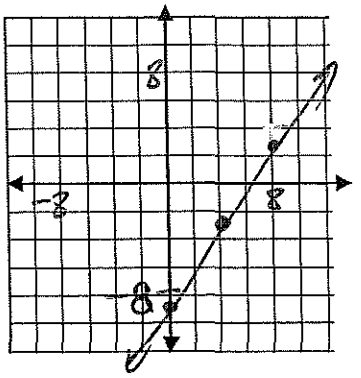
$y + 4 = 2(x - 3)$

***Making the Connection*: Relating slope-point, slope-intercept and the general form**

Ex2. Determine the equation of each line using slope-point form, slope-intercept form and the general form.

a) Given (–2 , 5), $m = 3$	b) Given (5, –1), $m = 3/2$	c) Given (8, –3), $m = -\frac{1}{2}$
Slope-point form: $y - y_1 = m(x - x_1)$ $y - 5 = 3(x + 2)$	$y + 1 = \frac{3}{2}(x - 5)$	$y + 3 = \frac{-1}{2}(x - 8)$
Slope-intercept form: $y = mx + b$ $y = mx + b$ Sub in (–2, 5) $5 = 3(-2) + b$ $11 = b$ so $y = 3x + 11$	$y + 1 = \frac{3}{2}x - \frac{15}{2}$ $y = \frac{3}{2}x - \frac{15}{2} - \frac{2}{2}$ $y = \frac{3}{2}x - \frac{17}{2}$	$y + 3 = -\frac{1}{2}x + \frac{8}{2}$ $y = -\frac{1}{2}x + 4 - 3$ $y = -\frac{1}{2}x + 1$
General Form: $Ax + By + C = 0$ $0 = 3x - y + 11$	$0 = \frac{3}{2}x - y - \frac{17}{2}$	$\frac{1}{2}x + y - 1 = 0$ $\uparrow \oplus$
Standard Form: $Ax + By = D$ $-11 = 3x - y$	$\frac{17}{2} = \frac{3}{2}x - y$	$\frac{1}{2}x + y = 1$

Ex3. Determine the equation of a line using two points.
Write an equation of the line through (3, –4) and (5, –1).
Then write the equation in the general form. Graph the equation.



Step 1. Given (3, –4) and (5, –1). Find slope (use slope formula).

$$m = \frac{-1-(-4)}{5-3} = \frac{3}{2}$$

Step 2. Since you have the slope and at least one point, use slope-point form, $y - y_1 = m(x - x_1)$ to write an equation (*Note: You can use either given point).* Change slope-point form to the general form: $Ax + By + C = 0$.

$$y - y_1 = m(x - x_1)$$
$$y + 4 = \frac{3}{2}(x - 3)$$
$$y + 4 = \frac{3}{2}x - \frac{9}{2}$$
$$y = \frac{3}{2}x - \frac{9}{2} - \frac{8}{2}$$
$$y = \frac{3}{2}x - \frac{17}{2}$$
$$0 = \frac{3}{2}x - y - \frac{17}{2}$$

Ex4. Determine if the equations are the same. (*Hint: Change to slope-intercept form.)*

a) $y - 5 = 2(x - 4)$

$y - 5 = 2x - 8$

$y = 2x - 3$

b) $y - 6 = 2(x - 10)$

$y - 6 = 2x - 20$

$y = 2x - 14$

c) $y + 9 = 2(x + 3)$

$y + 9 = 2x + 6$

$y = 2x - 3$

Try these:

Ex5. What is the *y*-intercept of a line with a slope of ½ and an *x*-intercept of 4? → (4, 0)

$y = \frac{1}{2}x + b$

$0 = \frac{1}{2}(4) + b$

$-2 = b$ So y int = (0, 2)

Ex6. Determine the *x*-intercept of a line through (3, 4) having a *y*-intercept of 2. (0, 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-0} = \frac{2}{3}$$

$y = \frac{2}{3}x + 2$ x-int (x, 0)

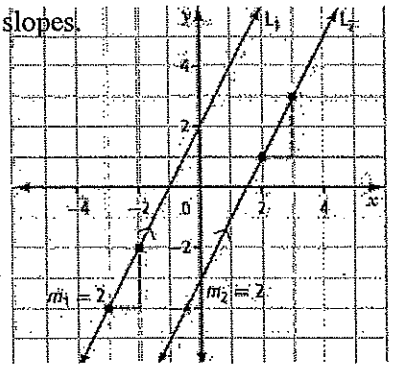
$0 = \frac{2}{3}x + 2$ $-2 = \frac{2}{3}x$

$-3 = x$

Mathematics 10, p377-381 (1, 2, 3, 4a, 5, 6ace, 7, 8, 11, 12, 14, 17)

Math 10 Name_____
Linear Equations and Graphs Date_____Blk___
7.4 **Parallel and Perpendicular Lines**

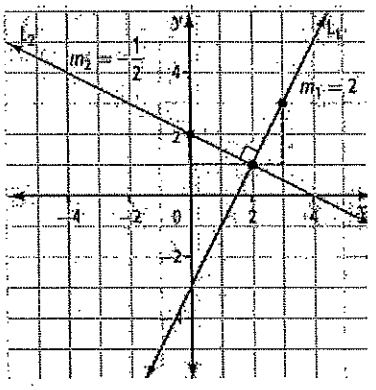
The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.



Parallel lines:

- Do not intersect
- have the **same slope** but **different intercepts**
- all horizontal lines, slope of zero, are parallel to each other
- all vertical lines, undefined slope, are also parallel to each other

Perpendicular lines:



- Two lines that intersect at right angles (90°)
- The **slopes of two lines that are negative reciprocals** of each other
- The product of negative reciprocals is –1.
- A vertical line (undefined slope) and horizontal line (0 slope) are perpendicular.

Ex1. Identify Parallel and Perpendicular Lines
State whether the lines in each pair are parallel, perpendicular or neither.

a) $y = 4x + 3$
$y = 4x - 5$

parallel

b) $y = 3x - 6$
$y = -2/3x + 4$

neither

c) $y = 2x + 6$
$6x + 3y + 3 = 0$

$3y = -6x - 3$

$y = \frac{-6}{3}x - 1$

$y = -2x - 1$

neither

d) $y = ½\,x - 7$
$y = -2x - 7$

perpendicular

e) $y = 3x - 4$
$y = 3x + 1/4$

parallel

f) $y = 2/5x - 6$
$5x + 2y = 8$

$y = -5x + 8$

$y = \frac{-5}{2}x + 4$

perpendicular

Write an Equation Involving a Parallel Line

Ex2. Write an equation parallel to:
$y = 3x - 4$ and passing through (2, 5)

Step 1. Determine **slope**, $m =$ 3

$y - y_1 = m(x - x_1)$

$y - 5 = 3(x - 2)$

Step 2. Given (2, 5) and slope, 3
Use slope-point form, then change to slope-intercept.

$y - 5 = 3x - 6$

$y = 3x - 1$

Ex3. Write an equation parallel to:
$2x - y + 4 = 0$ and passing through (1,-6)

Step 1. Find slope by changing equation to slope-intercept form
$2x - y + 4 = 0$

$2x + 4 = y$ slope = 2

Step 2. Given (1,-6) and slope, 2
Use slope-point form, then change to general form

$y - y_1 = m(x - x_1)$

$y + 6 = 2(x - 1)$

$y + 6 = 2x - 2$

$y = 2x - 8$

Write an Equation Involving a Perpendicular Line

Ex4. Write an equation perpendicular to:
$y = 3x - 4$ and through (6, 5)

Step 1. Determine **slope**, $m =$ 3 $-\frac{1}{3}$
The **negative reciprocal of slope** is 3.

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{3}(x - 6)$

Step 2. Given (6, 5); negative reciprocal slope $-\frac{1}{3}$
Use slope-point form, then change to slope-intercept.

$y - 5 = -\frac{1}{3}(x - 6)$

$y - 5 = -\frac{1}{3}x + \frac{6}{3}$

$y = -\frac{1}{3}x + 2 + 5$

$y = -\frac{1}{3}x + 7$

Ex5. Write an equation perpendicular to:
$2x - y + 4 = 0$ and through (1,-6)

Step 1. Find slope by changing equation to slope-intercept form
Then find negative reciprocal of slope $-\frac{1}{2}$.

$y = 2x + 4$

so $m = \frac{-1}{2}$

Step 2. Given (1,-6) and negative reciprocal slope, $-\frac{1}{2}$
Use slope-point form, then change to general form.

$y - y_1 = m(x - x_1)$

$y + 6 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + \frac{1}{2} - 6$

$y = -\frac{1}{2}x + \frac{1}{2} - \frac{12}{2}$

$y = -\frac{1}{2}x - \frac{11}{2}$

It's your turn.

Ex6. Write a **parallel line equation**
A line is parallel to $4x + y - 12 = 0$
and passes through (8, -6)

① Change to $y = mx + b$

$y = -4x + 12$

So the line has $m = -4$

$y - y_1 = m(x - x_1)$

$y + 6 = -4(x - 8)$

② Change to $y = mx + b$

$y + 6 = -4x + 32$

$y = -4x + 26$

Ex7. Write a **perpendicular line equation**
A line is perpendicular to $4x + y - 12 = 0$
and passes through (8, -6)

① Change to $y = mx + b$

$y = -4x + 12$

So the line has $m = \frac{1}{4}$

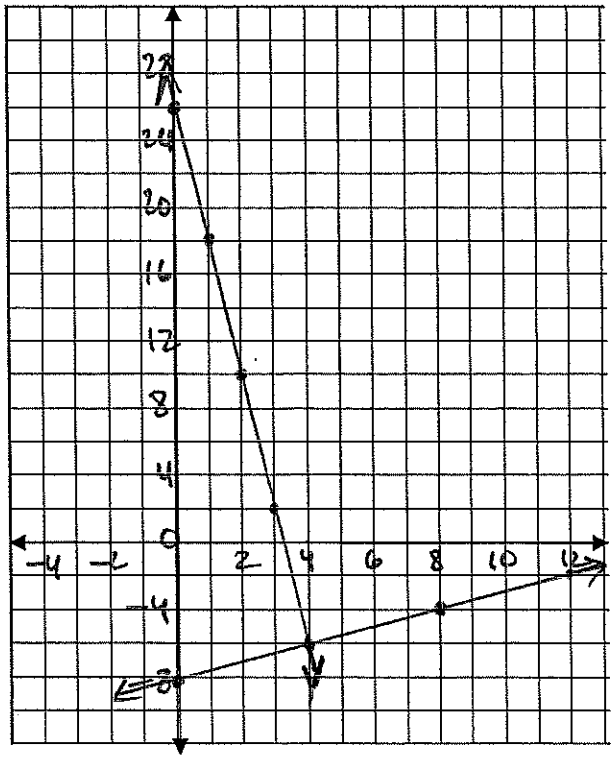
$y + 6 = \frac{1}{4}(x - 8)$

② Change to $y = mx + b$

$y + 6 = \frac{1}{4}x - 2$

$y = \frac{1}{4}x - 8$

Graph and label the original line (L1), parallel line (L2), and perpendicular line (L3) for ***Ex6*** and **Ex7** above.



Ex8. For each pair of lines, determine if the lines are parallel.

a) Line A contains points (-9, 2) and (-3,4)
Line B contains points (-7, -7) and (1, -3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_A = \frac{4-2}{-3-(-9)} = \frac{2}{6} = \frac{1}{3}$

$m_B = \frac{-3-(-7)}{1-(-7)} = \frac{4}{8} = \frac{1}{2}$

No!

b) Line C contains points (-4,5) and (-2,-1)
Line D contains points (3,3) and (6, -3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_A = \frac{5-(-1)}{-4-(-2)} = \frac{6}{-2} = -3$

$m_B = \frac{3-(-3)}{3-6} = \frac{6}{-3} = -2$

No!

Ex9. Two lines are **parallel**. Determine k.
Line P contains points (-2,-4) and (-1,-1)
Line Q contains points (6,-2) and (3,k)

$m_P = \frac{-1-(-4)}{-1-(-2)} = \frac{3}{1} = 3$

$m_Q = \frac{k-(-2)}{3-6} = \frac{k+2}{-3}$

$\frac{k+2}{-3} = \frac{3}{1}$ (cross: $\times(-3)$, $\times(-3)$)

So $-9 = k+2$

$-11 = k$

Ex10. Two lines are **perpendicular**. Determine k.
Line P contains points (-2,-4) and (-1,-1)
Line Q contains points (6,-2) and (3,k)

$m_Q = \frac{k-(-2)}{3-6} = \frac{k+2}{-3}$

slope is $-\frac{1}{3}$

$\frac{k+2}{-3} = -1$

$-3 = 3$ ($\times(-1)$)

$k+2 = 1$

$\boxed{k = -1}$

Ex11. Find the equation of a line perpendicular to $2x + 5y + 10 = 0$ with the same x-intercept as $3x - 2y = 12$.

① Change to $y = mx + b$.

$2x + 5y + 10 = 0$

$5y = -2x - 10$

$y = -\frac{2}{5}x - 2$

So $m = \frac{5}{2}$

② Find x-int:

$3x - 2(0) = 12$

$3x = 12$

$\boxed{x = 4}$

③ Sub (4,0) into the line

$y = \frac{5}{2}x + b$

$0 = \frac{5}{2}(4) + b$

$-10 = b$

So $y = \frac{5}{2}x - 10$

Mathematics 10, p390-394 (1 to 7, 9, 10, 11, 13, 14, 15, 16)